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Marginal stability of parallel whistler-mode waves (asymptotic analysis)

S. S. Sazhin [1] **and N. M. Temme** [2]

[1] Department of Physics, The University of Sheffield, Sheffield S3 7RH, UK
[2] Centrum voor Wiskunde en Informatica, Kruislaan 413, 1098 SJ, Amsterdam, The Netherlands

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Abstract. A new approach to the asymptotic analysis of the marginal stability of whistler-mode waves in a weakly relativistic plasma allows us to generalise the results of our previous papers to a wider range of parameters. In particular, it is pointed out that a decrease in electron density tends to stabilize whistler-mode instability, while an increase in electron temperature destabilizes or stabilizes it depending on the choice of the plasma model. However, the second effect is usually small when compared with the first one and can be neglected when analysing the dynamics of whistler-mode waves in the Earth's magnetosphere, as well as in the magnetospheres of other planets.

Introduction

In our previous papers (Sazhin and Temme, 1990, 1991) we presented an asymptotic analysis of relativistic effects on parallel whistler-mode propagation and instabilities. Our results appeared to be consistent with the previous simplified analytical analysis by Sazhin (1989) and were shown to be potentially useful for the interpretation of whistler-mode dynamics in the magnetosphere of the Earth. The analysis of our papers was based on the dispersion equation for parallel whistler-mode waves written in the form:

$$N^2 = 1 - \frac{2X}{r}\left[\mathscr{F}_{1/2,2} - \frac{d\mathscr{F}_{3/2,2}}{dz}(A_e - 1)N^2\right], \tag{1}$$

where

$$\mathscr{F}_{q,p} \equiv \mathscr{F}_{q,p}(z,a,b) = -i\int_0^\infty e^{izt - \frac{at^2}{1-it}}(1-it)^{-q}(1-ibt)^{-p}\,dt, \tag{2}$$

$$z = \frac{2(1-Y)}{r};\ a = \frac{N^2}{r};\ r = \frac{p_{0\|}^2}{m_e^2 c^2};\ b = A_e;\ X = \Pi_0^2/\omega^2;$$

$$Y = \Omega_0/\omega;\ A_e = p_{0\perp}^2/p_{0\|}^2;$$

Π_0, Ω_0 and ω are electron plasma frequency at rest, electron gyrofrequency at rest and wave frequency respectively; N is the wave refractive index.

When deriving Eq. 1 we assumed an electron distribution function of the form:

$$f(p_\perp, p_\|) = (j!\,\pi^{3/2}\,p_{0\perp}^{2j+2}\,p_{0\|})^{-1}\,p_\perp^{2j}\exp\left(-\frac{p_\perp^2}{p_{0\perp}^2} - \frac{p_\|^2}{p_{0\|}^2}\right), \tag{3}$$

where $p_{0\perp(\|)}$ is the electron thermal momentum in the direction perpendicular (parallel) to the magnetic field, $p_\perp$ and $p_\|$ are the electron momenta in the corresponding directions, and $j = 0, 1, 2, \ldots$ We restricted our analysis to the case $j = 0$ (bi-Maxwellian plasma). The generalization to $j \neq 0$ is straightforward (Tsai *et. al.*, 1981; Sazhin, 1989). We also restricted ourselves to considering wave frequencies well above the proton gyrofrequency so that the protons and other heavy ions could be considered as a neutralizing background. Finally, we assumed the plasma to be weakly relativistic so that $p_{0\perp(\|)} \ll m_e c$, where m_e is the electron mass at rest and c is the velocity of light.

The function $\mathscr{F}_{q,p}$ was called the generalized Shkarofsky function by analogy with the conventional Shkarofsky function $\mathscr{F}_q(z, a)$, defined e.g. by Robinson (1986, 1987):

$$\mathscr{F}_q(z, a) = -i\int_0^\infty e^{izt - \frac{at^2}{1-it}}(1-it)^{-q}\,dt \tag{4}$$

[see also the original paper by Shkarofsky (1966)].

We have the straightforward identities:

$$\mathscr{F}_{q,0}(z, a, b) = \mathscr{F}_{q,p}(z, a, 0) = \mathscr{F}_q(z, a),$$

$$\mathscr{F}_{q,p}(z, a, 1) = \mathscr{F}_{q+p}(z, a).$$

Both the real and the imaginary parts of Eq. 1 were analysed by Sazhin and Temme (1990) in the asymptotic limit $a \to \infty$ and $|\mu| = |z/a| \ll 1$, while Sazhin and Temme (1991) restricted their analysis to the imaginary part of Eq. 1 in the limit $|A_e - 1| \ll 1$. The analysis of the imaginary part of this equation was of particular interest as it allowed us to consider the problem of the stability of the

Offprint requests to: S. S. Sazhin

waves. In this paper we concentrate our attention just on this very aspect of the problem. We begin with an analysis of the imaginary part of $\mathscr{F}_{q,p}$ based on the ideas of Sazhin and Temme (1990) but using more terms in the asymptotic expansion of $\Im\mathscr{F}_{q,p}$ (second section). Based on these results and Eq. 1 we then consider the problem of marginal stability of the waves. Here again we extend the results of Sazhin and Temme (1990, 1991) taking into account higher order relativistic corrections to the value of A_e when the waves are marginally stable. In contrast to Sazhin and Temme (1990) we impose a milder restriction on μ (not $|\mu| \ll 1$ but $\mu < 1$). In contrast to Sazhin and Temme (1991) we impose no specific restrictions on the value of A_e, although we assume that $|a| \gg 1$. The later inequality always holds true in the Earth's magnetosphere. Our results are important not only from the point of view of obtaining the higher order correction themselves but also from the point of view of justifying the applicability of the lower order approximation when only the first order corrections are taken into account. The main conclusions of the paper are summarized in the final section.

Asymptotics for $\Im\mathscr{F}_{q,p}$

As was shown by Sazhin and Temme (1990), in the limit $a \to \infty$ and $\mu = z/a < 1$ we can write:

$$\Im\mathscr{F}_{q,p} \sim \Im\left\{ e^{-ax_0^2} \sum_{k=0}^{\infty} c_k \int_0^{\infty} e^{-4a\sqrt{1-\mu}\,\omega^2} \omega^k \, d\omega \right\}$$

$$= \Im\left\{ \tfrac{1}{2} e^{-ax_0^2} \sum_{k=0}^{\infty} c_k \Gamma\left(\frac{k+1}{2}\right) [4a\sqrt{1-\mu}]^{-(k+1)/2} \right\}, \quad (5)$$

where the c_k are defined as:

$$f(s) \frac{ds}{d\theta} \frac{d\theta}{d\omega} = \sum_{k=0}^{\infty} c_k \omega^k, \quad (6)$$

$$\omega = -\sin \tfrac{1}{2}\theta, \quad (7)$$

$$s = -1 + \frac{\cos\theta + i \sin\theta}{\sqrt{1-\mu}}, \quad (8)$$

$(-\pi \leq \theta \leq 0)$, with:

$$f(s) = (1+s)^{-q} (1+bs)^{-p}, \quad (9)$$

and

$$x_0 = \sqrt{2 - \mu - 2\sqrt{1-\mu}}. \quad (10)$$

Based on the definitions of s and ω we can write:

$$s = s_+ + \frac{1}{\sqrt{1-\mu}} \left[-2i\omega - 2\omega^2 + i\omega^3 + \frac{i\omega^5}{4} + \dots \right], \quad (11)$$

where

$$s_+ = -1 + \frac{1}{\sqrt{1-\mu}}.$$

Furthermore, we need:

$$\frac{ds}{d\theta} \frac{d\theta}{d\omega} = \frac{ds}{d\omega} = \frac{1}{\sqrt{1-\mu}} \left[-2i - 4\omega + 3i\omega^2 + \frac{5i\omega^4}{4} + \dots \right] \quad (12)$$

and

$$f(s) = f(s_+) [1 + f_1 \omega + f_2 \omega^2 + \dots], \quad (13)$$

where

$$f_1 = 2i \frac{\sqrt{1-\mu}\,(b-1)q - b(q+p)}{\sqrt{1-\mu}\,(b-1) - b},$$

$$f_2 = 2\left\{ \frac{\sqrt{1-\mu}\,(b-1)\,b(2q^2+2qp-p)}{[\sqrt{1-\mu}\,(b-1)-b]^2} + \frac{(1-\mu)[2q^2b - q^2] + q^2b^2(\mu-2) - b^2[p^2+2pq]}{[\sqrt{1-\mu}\,(b-1)-b]^2} \right\}.$$

Having substituted Eqs. 12 and 13 into expansion 6 we obtain the following expressions for the first three coefficients in this expansion

$$c_0 = \frac{-2i}{\sqrt{1-\mu}} f(s_+), \quad (14)$$

$$c_1 = \frac{-4[\sqrt{1-\mu}\,(b-1)(1-q) + b(q+p-1)]}{\sqrt{1-\mu}\,[\sqrt{1-\mu}\,(b-1)-b]} f(s_+), \quad (15)$$

$$c_2 = \frac{-i[\tilde{a} + \tilde{b}\mu + \tilde{c}\sqrt{1-\mu}]}{\sqrt{1-\mu}\,[\sqrt{1-\mu}\,(b-1)-b]^2} f(s_+), \quad (16)$$

where

$$\tilde{a} = b^2(-8q^2 - 4p^2 - 8qp + 16q + 8p - 6) + (2b-1)(4q^2 - 8q + 3),$$

$$\tilde{b} = (b-1)^2 (4q^2 - 8q + 3),$$

$$\tilde{c} = 2b(b-1)(4q^2 + 4qp - 8q - 6p + 3).$$

Sazhin and Temme (1990) derived only the coefficients c_0 and c_1. The coefficient c_1 contributes to the $\Re\mathscr{F}_{q,p}$ and was used for the analysis of whistler-mode propagation, while the coefficient c_0 contributes to the $\Im\mathscr{F}_{q,p}$ and was used by Sazhin and Temme (1990) for the analysis of marginal stability of the waves. However, this part of their analysis was not strictly rigorous as the neglection of the term c_2 (which also contributes to $\Im\mathscr{F}_{q,p}$) was not justified. Moreover, they imposed the condition $|\mu| \ll 1$ which is sometimes rather stringent for applications. We will derive our main equations under the milder restriction $\mu < 1$.

Having substituted Eqs. 14 and 16 into Eq. 5 we obtain:

$$\Im\mathscr{F}_{q,p} \sim -\frac{\sqrt{\pi}\, f(s_+) \exp(-ax_0^2)}{2\sqrt{a}\,(1-\mu)^{3/4}} \times \left[1 + \frac{\tilde{a} + \tilde{b}\mu + \tilde{c}\sqrt{1-\mu}}{16a(1-\mu)^{5/4}\,[\sqrt{1-\mu}\,(b-1)-b]^2} \right], \quad (17)$$

where $\tilde{a}$, $\tilde{b}$ and $\tilde{c}$ are the same as in Eq. 16.

In the next section this expression will be used for an analysis of the condition of marginal stability of whistler-mode waves.

Analysis of marginally stable waves

Similarly to Sazhin and Temme (1990) we can present the condition for marginal stability of the waves as:

$$\Im\mathscr{F}_{1/2,2}=\frac{d\Im\mathscr{F}_{3/2,2}}{dz}(A_e-1)N^2. \tag{18}$$

When Eq. 18 is valid, the imaginary part of the right hand side of Eq. 1 is equal to zero.

Having substituted Eq. 17 into Eq. 18 we obtain the equation of marginal stability in a more explicit form:

$$(b-1)N^2 f_{3/2}(s_+)\left[1+\frac{\tilde{a}_{3/2}+\tilde{c}_{3/2}\sqrt{1-\mu}}{16a(1-\mu)^{5/4}[\sqrt{1-\mu}(b-1)-b]^2}\right]$$
$$=[1+(b-1)N^2]\cdot f_{1/2}(s_+)\left[1+\frac{\tilde{a}_{1/2}+\tilde{c}_{1/2}\sqrt{1-\mu}}{16a(1-\mu)^{5/4}[\sqrt{1-\mu}(b-1)-b]^2}\right], \tag{19}$$

where:

$\tilde{a}_{1/2(3/2)}=\tilde{a}(q=1/2(3/2)),\ \tilde{c}_{1/2(3/2)}=\tilde{c}(q=1/2(3/2)),$

$f_{1/2(3/2)}=f(q=1/2(3/2)).$

Remembering our assumption that a is large, we can expand A_e with respect to a^{-1} and write it as:

$$A_e=\tilde{A}_{e0}+\frac{\tilde{A}_{e1}}{a}+\frac{\tilde{A}_{e2}}{a^2}+\ldots \tag{20}$$

Having substituted Eq. 20 into Eq. 19 we obtain the following expressions for $\tilde{A}_{e0}$ and $\tilde{A}_{e1}$:

$$\tilde{A}_{e0}=1+\frac{1}{N^2[\sqrt{1-\mu}-1]}, \tag{21}$$

$$\tilde{A}_{e1}=-\frac{N^2[\sqrt{1-\mu}-1]+\mu N^2-\sqrt{1-\mu}}{N^2(1-\mu)^{5/4}[(N^2-1)(1-\mu)^{3/2}+N^2(3\sqrt{1-\mu}+3\mu-4)+4-3\mu]}. \tag{22}$$

The expression for A_{e2} is much more complicated and is only significant when we take into account one further term in Eq. 5.

At first sight expression 21 appears to contradict the expression 19 of Sazhin and Temme (1991) which in the limit $a\to\infty$ reduces to:

$$A_e=1+\frac{1}{(N^2-2)[\sqrt{1-\mu}-1]}. \tag{23}$$

However Eq. 23 was derived under the assumption that $|A_e-1|\ll 1$ which is satisfied only when $N^2\gg 1$. In this case we can neglect the 2 when compared with N^2 in the denominator of Eq. 23 and reduce Eq. 23 to Eq. 21 taken in the limit $|A_e-1|\ll 1$.

In the limit $|\mu|\ll 1$ Eqs. 21–22 reduce to:

$$\tilde{A}_{e0}=-\frac{2}{\mu N^2}+1+\frac{1}{2N^2}+\frac{\mu}{8N^2}+\ldots, \tag{24}$$

$$\tilde{A}_{e1}=-\frac{1}{8N^2(N^2-1)\mu^3}+\frac{4}{(N^2-1)\mu^2}+\frac{4N^2-3}{4\mu N^2(N^2-1)}+\ldots. \tag{25}$$

Taking the first three terms in Eq. 24 we can write it in a more explicit form:

$$\tilde{A}_{e0}=\frac{Y}{Y-1}+\frac{1}{2N^2}. \tag{26}$$

Equation 26 is equivalent to the corresponding expression derived by Sazhin and Temme (1990) in the limit $N^2\gg z^2(Y-1)/4a\gg 1$ (see their Eqs. 5.3 and 5.6. However, the other expression for A_e derived by Sazhin and Temme (1990) (see their Eqs. 5.3 and 5.8) in the limit $z^2(Y-1)/4a\gg N^2\gg 1$ has no physical meaning, as in this limit the condition $\mu z/4a\to 0$, on which the analysis of Sazhin and Temme (1990) was based, cannot be satisfied.

In what follows we will return to Eqs. 21–22 valid for $\mu<1$.

The general expression for N^2 for whistler-mode waves, with thermal and relativistic corrections taken into account, can be written as (Jaquinot and Leloup, 1971; Sazhin 1987 a, b):

$$N^2=N_0^2\left[1+\left(\frac{vY^3}{2(Y-1)^3}+\frac{v(1+4A_e)Y^2}{4(Y-1)^2N_0^2}-\frac{vA_eY^2}{2(Y-1)^2}\right)r\right], \tag{27}$$

where $v=\Pi_0^2/\Omega_0^2$,

$$N_0^2=1+\frac{vY^2}{Y-1}. \tag{28}$$

Restricting ourselves to considering frequencies at which the waves are marginally stable, we can put in Eq. 27 $A_e=Y/(Y-1)$ and simplify it to:

$$N^2=N_0^2\left[1+\frac{v(5Y-1)Y^2r}{4(Y-1)^3N_0^2}\right]. \tag{29}$$

Having substituted Eq. 29 into Eqs. 21–22 and neglecting higher order terms with respect to r we can rewrite Eq. 20 as:

$$A_e=A_{e0}+A_{e1}r, \tag{30}$$

where

$$A_{e0}=1+\frac{1}{N_0^2[\sqrt{1-\mu_0}-1]}, \tag{31}$$

$$A_{e1}=-\frac{N_0^2[\sqrt{1-\mu_0}-1]+\mu_0N_0^2-\sqrt{1-\mu_0}}{N_0^4(1-\mu_0)^{5/4}[(N_0^2-1)(1-\mu_0)^{3/2}+N_0^2(3\sqrt{1-\mu_0}+3\mu_0-4)+4-3\mu_0]}$$
$$-\frac{v(5Y-1)Y^2}{4N_0^4(\sqrt{1-\mu_0}-1)(Y-1)^3}\left[1+\frac{\mu_0}{2(\sqrt{1-\mu_0}-1)\sqrt{1-\mu_0}}\right], \tag{32}$$

$$\mu_0=2(1-Y)/N_0^2.$$

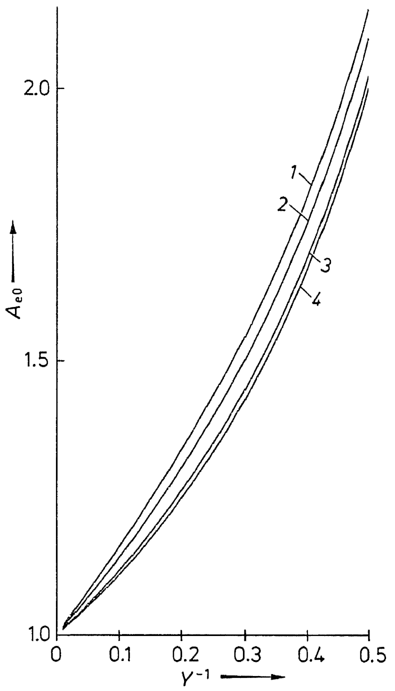

Fig. 1. Plots of A_{e0} versus Y^{-1} for $v=\Pi_0^2/\Omega_0^2=0.5$ (*curve* 1), $v=1$ (*curve* 2), $v=5$ (*curve* 3) and $v=100$ (*curve* 4)

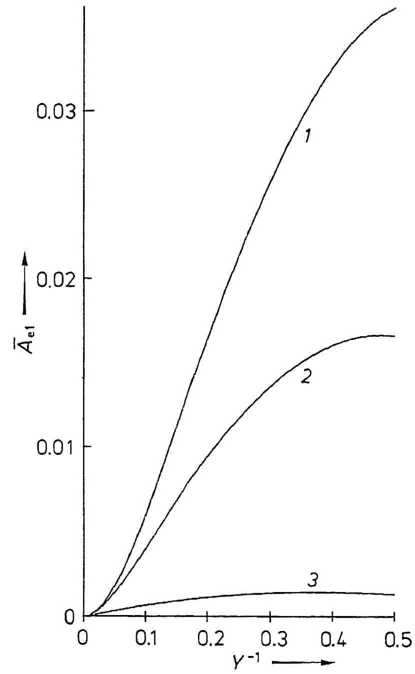

Fig. 3. As in Fig. 1 but for $\bar{A}_{e1}$. The curve corresponding to $v=100$ almost coincides with Y^{-1} axis

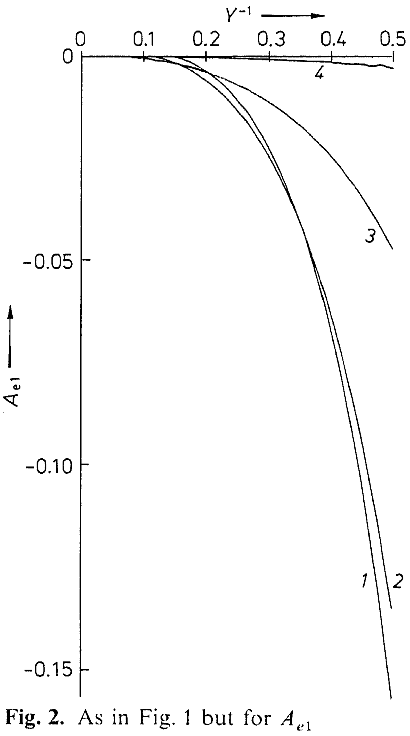

Fig. 2. As in Fig. 1 but for A_{e1}

Plots of A_{e0} versus Y^{-1} and A_{e1} for the same v as in Sazhin and Temme (1991) are shown in Figs. 1 and 2 respectively. We consider $Y^{-1}<0.5$ when A_{e0} is about or below 2 (typical values of this parameter in the Earth's magnetosphere, see e.g. Tsurutani *et al.*, 1982; Bahnsen *et al.*, 1985; Solomon *et al.*, 1988). Also, for the values of the parameters Y^{-1} and v under consideration, the condition $\mu<1$, on which our theory is based, is satisfied. As follows from Fig. 1, a decrease in electron density (parameter v) tends to stabilize whistler-mode waves in agreement with the earlier results of Sazhin and Temme (1991), who reached the same conclusion for $Y^{-1}\ll 1$ and $|A_e-1|\ll 1$. In other words a decrease in electron density requires a larger anisotropy to achieve marginal stability.

At the same time, as follows from Fig. 2, and Eq. 30 the non-zero electron temperature (parameter r) tends to decrease $A_e(A_{e1}<1)$ thus destabilizing whistler-mode waves (smaller anisotropy is required to achieve marginal stability). However, in the actual conditions of the Earth's magnetosphere, when r is almost always below 10^{-2} (electron energy below several keV), this stabilization of whistler-mode instability appears to be negligibly small.

Equation 20 can also be used for the analysis of whistler mode instability in a slightly different plasma model when we assume that the density of the hot electrons responsible for whistler-mode instability is well below the density of the "cold" electrons responsible for whistler-mode propagation (cf. Kennel and Petschek, 1966; Etcheto *et al.*, 1973). In this case we can assume that $N=N_0$ and rewrite Eq. 30 as:

$$A_e=A_{e0}+\bar{A}_{e1}\,r\,, \tag{33}$$

where A_{e0} is determined by Eq. 31,

$$\bar{A}_{e1}=\tilde{A}_{e1}(N=N_0)/N_0^2 \tag{34}$$

($\tilde{A}_{e1}$ is determined by Eq. 22).

As follows from the plot $\bar{A}_{e1}$ versus Y^{-1} (Fig. 3), in this case the effect of non-zero electron temperature (non-zero r) tends to stabilize whistler-mode waves. However, the efficiency of this process appears to be even smaller than in the case of destabilization due to $A_{e1}<0$.

Finally, we attempted to compare the results shown in Fig. 1 with the results of numerical analysis of Jacquinot and Leloup (1971). In fact, Jacquinot and Leloup considered not the problem of marginal stability of the waves but the frequency dependence of their increment of instability (γ). However, it seems possible to get the values of Y^{-1} for marginally stable waves from the curves given in their Fig. 5 if we extrapolate them to the point where $\gamma=0$. Having done this, we obtained that at $A_e=2$ and $\nu=1$ relativistic effects decrease the frequency of marginally stable waves from $Y^{-1}=0.5$ to $Y^{-1}\approx 0.42$, while our Fig. 1 predicts a decrease from $Y^{-1}=0.5$ to $Y^{-1}\approx 0.47$ at the same A_e and ν. The reason for this apparent discrepancy between our results is not clear, but we suspect that it might have its origin in the low accuracy of extrapolation of the corresponding curve of Jacquinot and Leloup (1971).

Main conclusions

The conclusion of Sazhin and Temme (1991) that whistler-mode waves are stabilized by a decrease in electron density is confirmed without the restriction that $|A_e-1|\ll 1$.

If we assume that the electron distribution is of the type described by Eq. 3 then an increase in the electron temperature tends to destabilize whistler-mode waves. However, if we assume that hot electrons with the distribution 3 influence whistler-mode instability or damping, while wave propagation is mainly determined by cold electrons with a density well above that of hot electrons, then an increase in r leads to an opposite effect. In both cases the effect of non-zero electron temperature on marginally stable waves appears to be much smaller than the effect of a decrease in electron density. This justifies the application of expression 31 to the analysis of marginally stable whistler-mode waves.

Our results are potentially most useful for the analysis of marginal stability of whistler-mode waves in those regions of the Earth's magnetosphere and the magnetospheres other planets (e.g. Neptune) where the plasma is relatively rarefied ($\nu \leq 1$) (see e.g. Curtis, 1978; Gurnett *et al.*, 1990).

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References

Bahnsen, A., M. Jespersen, T. Neubert, P. Canu, H. Borg, and P. E. Frandsen, Morphology of keV-electrons in the Earth's magnetosphere as observed by GEOS-1, *Ann. Geophysicae,* **3,** 19–25, 1985.

Curtis, S. A., Theory for chorus generation by energetic electrons during substorms, *J. Geophys. Res.,* **83,** 3841–3848, 1978.

Etcheto, J., R. Gendrin, J. Solomon, and A. Roux, A self-consistent theory of magnetospheric ELF hiss, *J. Geophys. Res.,* **78,** 8150–8166, 1973.

Gurnett, D. A., W. S. Kurth, I. H. Cairns, and L. J. Granroth, Whistlers in Neptune's magnetosphere: evidence of atmospheric lightning, *J. Geophys. Res.,* **95,** 20 967–20 976, 1990.

Jacquinot, J., and C. Leloup, Electron cyclotron electromagnetic instabilities in weakly relativistic plasma, *Phys. Fluids,* **14,** 2440–2446, 1971.

Kennel, C. F., and H. E. Petschek, Limit of stably trapped particle fluxes, *J. Geophys. Res.,* **71,** 1–28, 1966.

Robinson, P. A., Relativistic plasma dispersion functions, *J. Math. Phys.,* **27,** 1206–1214, 1986.

Robinson, P. A., Relativistic plasma dispersion functions: Series, integrals, and approximations, *J. Math. Phys.,* **28,** 1203–1205, 1987.

Sazhin, S. S., An approximate theory of electromagnetic wave propagation in a weakly relativistic plasma, *J. Plasma Phys.,* **37,** 209–230, 1987a.

Sazhin, S. S., A physical model of parallel whistler-mode propagation in a weakly relativistic plasma, *J. Plasma Phys.,* **37,** 301–307, 1987b.

Sazhin, S. S., Parallel whistler-mode propagation in a weakly relativistic plasma, *Phys. Scripta,* **40,** 114–116, 1989.

Sazhin, S. S., and N. M. Temme, Relativistic effects on parallel whistler-mode propagation and instability, *Astrophys. Space Sci.,* **166,** 301–313, 1990.

Sazhin, S. S., and N. M. Temme, The threshold of parallel whistler-mode instability, *Ann. Geophysicae,* **9,** 30–33, 1991.

Shkarofsky, I. P., Dielectric tensor in Vlasov plasmas near cyclotron harmonics, *Phys. Fluids,* **9,** 561–570, 1966.

Solomon, J., N. Cornilleau-Wehrlin, A. Korth, and G. Kremser, An experimental study of ELF/VLF hiss generation in the Earth's magntosphere, *J. Geophys. Res.,* **93,** 1839–1847, 1988.

Tsai, S. T., C. S. Wu, Y. D. Wang, and S. W. Kang, Dielectric tensor of a weakly relativistic, nonequilibrium, and magnetized plasma, *Phys. Fluids.* **24,** 2186–2190, 1981.

Tsurutani, B. T., E. J. Smith, R. R. Anderson, K. W. Ogilivie, J. D. Scudder, D. N. Baker, and S. J. Bame, Lion roars and nonoscillatory drift mirror waves in the magnetosheath, *J. Geophys. Res.,* **87,** 6060–6072, 1982.