B11

Michiel Hazewinkel
Direct line: +31-20-5924204
Secretary: +31-20-5924233
Fax: +31-20-5924166
E-mail: mich@cwi.nl

CWI
POBox 94079
1090GB Amsterdam

original version: 5 May 2002
revised version: 10 May 2002

Benjamin H Yandell, The honors class. Hilbert's problems and their solvers, A K Peters, 2002, 486 pp, £28.00, ISBN 1-56881-141-1, hbk.

This book is about Hilbert's 23 problems from the Paris ICM in 1900. It discusses both the problems themselves at a semi-popular level, but mostly it is about the people who tried to solve them or who were involved one way or another. And, as the book amply proves, that is a very rich source of material, for by and large mathematicians, certainly the really good ones, are most colourful and interesting people.

The author, Benjamin Yandell, is an amateur in both meanings of the word. He is not (really) a professional mathematician and he most obviously loves his subject. He has followed the excellent method of consulting the experts time and time again till he was absolutely sure that he had things right.[1] I expect that some of his victims in this got heartily tired of him. Some have given Yandell alot of time (and consequently figure more prominently than others).The result is worth the trouble Yandell and his sources took. This is a fascinating book to read and it contains a wealth of anecdotal and biographic material that must have taken great effort to collect (though perhaps less than one might think because the favourite topic of conversation when mathematicians get together are the vagaries of, and stories about, other mathematicians).

All in all I think this is an excellent effort, and much of the more rewarding material is in the semitechnical description of the real mathematics involved. Easy reading this book is definitely not; as the author recommends, read it like a scientist would; don't worry if something is for the moment ununderstandable; there is always a good chance that things will become clearer later, once your brain has had time to ruminate and dream a bit.

For this review I will concentrate on the foundational and logical problems. That is problems 1 (Cantor's problem on the cardinal of the continuum, more colloquially known as the continumm hypothesis (CH)), 2 (the compatibility of the arithmetical axioms), and 10 (Determination of the solvability of a Diophantine equation), respectively.

As already indicated, besides a wealth of anecdotal and biographical material the author devotes a lot of space to a semitechnical description of what the problems are about, what the motivations were (or could have been), and what the solutions look like. This is particularly the case with these foundational problems. The reader really gets some feeling for what is involved and why things have turned out as they did. That is, he gets some, perhaps only intuitive, idea of what the Gödel incompleteness theorems are and what they imply and what Cohen forcing is. Personally, like most professional mathematicians I would think, I still prefer more technical treatments[2], but that is probably a function of training and personal inclinations. There is no doubt the author has done quite a good job. I found the account of the tenth problem particularly nicely done.

Not all problems are treated as well. For instance, the account of the twenty first problem is not particularly enlightening. Quite generally the 'anaysis problems' (13, 19-23) get rather less space in the book, with the 23-rd least of all (half a page and no anecdotal of biographical material. The author provides a thoughtful remark on this, not entirely without gumption[3]:

"First, the structure of analysis is less clean,... . Second, as of 1900 Hilbert had not been an analist long—he wasn't really ready to write these problems. His analysis problems are less sharply drawn than the problems in the other areas."

---

[1] For a rather concentrated survey of the current state of affairs of Hilbert's problems see the article 'Hilbert problems' in M Hazewinkel (ed), Encyclopaedia of Mathematics, Supplement II (= volume 12), Kluwer Academic Publishers, 2000. It is nice to be able to remark that the census in the book under review, p. 385ff, by and large agrees completely with the one of loc. cit.

[2] See, for instance the excellent booklet by E Nagel and J Newman, Gödel's proof, Routledge and Kegan Paul, 1959, and since then translated and reissued numerous times. Oddly, this little jewel does not occur in the references of Yandell's book.

[3] See p.295ff.

Michiel Hazewinkel
Direct line: +31-20-5924204
Secretary: +31-20-5924233
Fax: +31-20-5924166
E-mail: mich@cwi.nl

CWI
POBox 94079
1090GB Amsterdam

original version: 5 May 2002
revised version: 10 May 2002

I most definitely agree. Fortunately analysis is becoming more and more algebraic (and combinatorial, and also logic is making major contributions to analyis nowadays); so there is still hope.

Thus the analysis problems get comparatively less space in this book[4] [5]. That is in fact the case in every account of Hilbert's problems that I have seen. In spite of the fact that an enormous amount of work has been done in this century, particularly in the direction of making analysis less a dirty subject.

No matter. This is a very worthwhile book to buy and read, and those that have the courage and stamina to do so will be richly rewarded.

---

[4] But the 22-nd problem with Koebe and Poincaré as main players gets a nice treatment. Of course many would not really count this field as analysis anymore. The theory of functions of one or more complex variables, that is analytic geometry, gets more and more the flavour of algebraic geometry (and, hence, algebra). In my view, in these sections, that autofamous person, Paul Koebe, get's a nicer treatment than may be correct. Interested persons are invited to consult the relevant sections of the biography of L E J Brouwer by D van Dalen, Oxford University Press, 1999.

[5] The positive solution by Kolmogorov and Arnol'd of the second half (the 'analysis' half; the other half is algebraic and still unsolved) says that every continuous function of n variables can be written as a composite (superposition) of continuous functions of two variables. I would see this rather as some kind of combinatorics. Part of the point being that things change drastically if differentiability or analyticity conditions are imposed.