

repulsive interactions matter more, the models (are expected to) exhibit much richer behaviour. In particular, it is expected (and has partly been proved) that the Domb-Joyce model exhibits the following qualitatively different behaviour depending on the value of the parameter  $\beta$ .

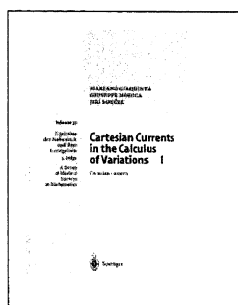
- If we fix  $\beta > 0$ , it shows a *ballistic* behaviour.
- On the other hand, if we decrease the value of  $\beta$  depending on  $n$  (the length of the walk) like  $\beta \approx n^{-3/2}$ , it shows a *diffusive* behaviour.
- Between these two limiting cases it will show something in between, but no proof had been given.

The main purpose of the book is to prove and refine some of the above expectations (and similar results for the Edwards model) as mathematical theorems.

In section 2, the author considers the Domb-Joyce model for fixed positive  $\beta$ . The main goal of this chapter is to give a detailed  $\beta$ -dependence of the speed constant and other important constants which characterize the ballistic behaviour. In section 3, the author considers the Edwards model with fixed  $\beta > 0$ . In section 4 (theorem 4.2 of the book), the author considers the Domb-Joyce model for  $n$ -dependent  $\beta_n$ , which satisfies  $\beta_n \rightarrow 0$  and  $\beta_n n^{3/2} \rightarrow \infty$ . Finally in section 5, the author obtains bounds on important constants appearing in chapters 2 through 4. Since the publication of the book more than three years ago, the author has made a lot more progress concerning one-dimensional random polymers. Some of the recent results can be found in a recent review article of the author.

In summary, this is a very nice book, and it can serve as a good introduction to the subject. Anyone interested in problems of one-dimensional random polymers are recommended to consult this book and the recent review article, before embarking on his own researches.

T. Hara



M. Giaquinta, G. Modica and J. Souček  
**Cartesian currents in the calculus of variations I and II**

(*Ergebnisse der Mathematik und ihrer Grenzgebiete; 37/38*)

Berlin: Springer-Verlag, 1998

711/697 p., prijs €234,-

ISBN 3-540-64009-6/3-540-64010-X

The title of this voluminous 2 volume monograph will convey little to most readers. No wonder, because the term *Cartesian currents* was coined by the authors only some 12 years ago. For that matter, even the term *current*, which dates back to René Thom, 1955, is not all that well known. Currents are to forms on a manifold as generalized functions (distributions in the sense of Schwartz) are to functions. More precisely a current is a functional on the linear space of compactly supported (alternating) forms on a manifold. It is homogeneous of dimension  $p$  if it is zero on all forms of degree unequal  $p$ . Thus currents can be seen as a kind of global distributions and they generalize both forms (covariant vectors) and vector fields (contravariant vectors). The name derives from the fact that currents of dimension 1 in three dimensional space can be interpreted as electric currents.

Cartesian currents are a special kind of current specifically de-

signed to handle nonscalar (global) variational problems such as minimal immersions, harmonic maps between Riemannian manifolds, and a variety of related problems in physics and mechanics. As is well known, optimal solutions often necessarily have singularities, and as in other fields a suitable notion of weak solutions needs to be defined. Very roughly Cartesian currents are those that can be (weakly) approximated by smooth (graphs of) maps. Here it is of considerable importance to take into account that in the nonscalar case the notion of a weak distributional derivative is not the right one to work with. Instead one needs to work with what is called (in these volumes) approximate differentiability.

The authors have made an enormous, and to my mind largely successful, effort to present this technically very complicated field in a self-contained and understandable way (though definitely high level; there is no way that these volumes can be seen as an introduction). For example they devote something like a hundred pages to explain the relevant parts of geometric measure theory (a notoriously difficult subject). They also have not hesitated, where appropriate, to do a bit of repetition to help the reader who does not want to go through all 1400 pages consecutively. There is no doubt that these two volumes should be on the shelves of just about every library with some pretense to be of value.

In a project of this size and level of sophistication there are inevitably some things that could have been better, some things to cavil at (which is, after all, one of the functions of a book reviewer). In this case, I noted four things: There are rather a lot of typos. There are quite a number of mangled/fractured sentences like (page 176 of Volume I): "A more complicate example which shows a lot of fractures in the graph is Cantor-Vitali function for which the approximate differential is zero almost everywhere and the graph in the previous sense consists of infinitely many horizontal segments, compare next chapter". There is some lack of motivational remarks. Not in the technical sense, more in the linguistic/philosophical sense. I would have liked to know, for example, why these particular currents are called *Cartesian* currents. Does this exhaustive, detailed two volume treatise belong in the *Ergebnisse* series. I think not. It should have been in the *Grundlehren* series both as regards style and type of book and level.

These are rather minor points. The authors are to be complimented on having done a magnificent job. A monograph like this illustrates well that also in the new electronic age books on paper will remain though journal papers may go more or less completely electronic.

M. Hazewinkel

J. Grasman en O.A. van Herwaarden  
**Asymptotic methods for the Fokker-Planck equation and the exit problem in applications**

(*Springer series in synergetics*)

Berlijn: Springer-Verlag, 1999

220 p., prijs €38,85

ISBN 3-540-64435-0

Uitgangspunt voor de auteurs is een niet-lineair dynamisch systeem voorzien van een kleine stochastische verstoring. De linearisatie met betrekking tot deze verstoring geeft de zogeheten Langevin vergelijking. Als  $p(t, x)$  de kansdichtheid aangeeft om het systeem in toestand  $x$  op tijd  $t$  te vinden, dan is  $p$  de oplossing van een lineaire diffusievergelijking, de zogenaamde Fokker-Planck-