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APPROXIMATION METHODS FOR NONLINEAR FILTERING PROBLEMS ARISING IN SYSTEM IDENTIFICATION:

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Abstract: In this paper we investigate various approximate methods for computing the conditional density of a parameter. These techniques are related to the structure of certain Lie algebras of operators with the identification problem.

Summary

Consider the stochastic differential system:

$$d\theta=0$$

$$dx_t = A(\theta)x_t dt + b(\theta)dw_t \tag{1}$$

$$dy_t = \langle c(\theta), x_t\rangle dt + dv_t.$$

Here $\{w_t\}$ and $\{v_t\}$ are independant, scalar, standard Wiener processes and $\{x_t\}$ is an $\mathbb{R}^n$-valued process. We let θ take values in a smooth manifold Θ $\mathbb{R}^n$. Assume that the map

$$\theta \mapsto \Sigma(\theta):=(A(\theta),\ b(\theta), c(\theta)) \tag{2}$$

is sufficiently smooth and takes values in the space of minimal triples.

Define two differential operators,

$$A_o := \tfrac{1}{2}\langle b(\theta),\ \partial/\partial x\rangle^2 - \langle \partial/\partial x,\ A(\theta)x\rangle - \langle c(\theta)\ x,\rangle^2/2 \tag{3}$$

$$B_o := \langle c(\theta),\ x\rangle. \tag{4}$$

The problem is to devise approximate finite dimensional, recursive techniques for calculating the conditional density of the parameter θ given Y_t = the σ-algebra generated by the observations $\{y_s: 0\leq s\leq t\}$. The general formulas are known:

$$Q(t,\theta) = \frac{\int \rho(t,x,\theta)\,|dx|}{\int\int \rho(t,x,\theta)\,|dx|\cdot|d\theta|} \tag{5}$$

where $|dx|$ and $|d\theta|$ are fixed Riemannian volume elements on $\mathbb{R}$ and Θ and

$$\rho(t,x,\theta) = e^{\langle c(\theta),x\rangle y_t}\,\psi(t,x,\theta) \tag{6}$$

and

$$\frac{\partial\psi}{\partial t} = \{\mathcal{L}_o + y_t\mathcal{L}_1 + \frac{y_t^2}{2}\mathcal{L}_2 + \mathcal{L}_3\}\psi \tag{7}$$

where

$$\mathcal{L}_o := A_o$$

$$\mathcal{L}_1 := \langle c(\theta), b(\theta)\rangle\langle b(\theta), \partial/\partial x\rangle - \langle c(\theta), A(\theta)x\rangle$$

$$\mathcal{L}_3 := \langle c(\theta),\ b(\theta)\rangle^2$$

$$\mathcal{L}_4 = -\mathrm{tr}\ (A(\theta)). \tag{8}$$

Let $Q(t,\theta) = e^{-S(t,\theta)}$. In this pair we consider approximations related to

(a) local series approximations

$$S(t,\theta) = \sum_{i=0}^{\infty} a_i(t)\ \theta^{[i]}$$

(b) Gaussian initial conditions:

$$\rho(0,.,\theta) \text{ Gaussian for } \theta\in\Theta$$

Both these approximations are connected to the following algebraic objects.

(a) A sequence of Lie algebras $\{G^{(k)}\}_{k=0}^{\infty}$ where

$$\tilde{G}^{(0)} := \{A_o, B_o\}_{L.A.}$$

$$\tilde{G}^{(1)} := \left\{\begin{bmatrix} A_o & 0 \\ \frac{\partial A_o}{\partial\theta} & A_o \end{bmatrix}, \begin{bmatrix} B_o & 0 \\ \frac{\partial B_o}{\partial\theta} & B_o \end{bmatrix}\right\}_{L.A.}$$

(b) Finite dimensional quotients of $\tilde{G}^{(0)}$ in one-to-one correspondence with rings that are quotients of $\mathbb{R}[\theta]$.

Our results use the fact that $\tilde{G}^{(0)}$ is a subalgebra of a current algebra ([1],[2]).

References

[1] P.S. Krishnaprasad and S.I. Marcus: "On the Lie Algebra of the identification problem", IFAC Symposium on Digital Control, New Delhi, Jan. 1982.

[2] P.S. Krishnaprasad, S.I. Marcus and M. Hazewinkel, "System identification and nonlinear filtering: Lie Algebras", Proc. 20th IEEE Conference on Decision and Control. San Diego, 1981.